Approximations of the Poisson equation using numerical methods: performance and results

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ABSTRACT

The modeling of natural phenomenas is very important for many areas of science and engineering, most of such models involves two or more independents variables, that results in a partial differential equation (PDE).

Between many partial differential equations, the Poisson equation, that is classified as an elliptical PDE, has one of the main papers in the modeling of incompressible viscous fluid flow. Such equation is result of algebraic manipulations of the momentum and continuity functions of the Navier-Stokes equation that govern this kind of fluid flow.

In the majority, after the problem has been shaped, the solution must be obtained by numerical methods. The resolution of the Poisson equation demands great computational effort, making very important to elect the method that has best cost-benefit between the approximations and the computational time.

Given the relevance of the subject, this work traced as objective, a survey of partial differential equations, with emphasis in elliptical PDEs, and the resolution of these equations using three kinds of numerical methods:

- Finite Difference Method (Five Points). With linear system solved by:

  Gauss Seidel

  Gauss Jacobi

  Successive Over Relaxation (SOR)

- Multi-Grid

- Alternating Direction Implicit (ADI)

After a general study of PDEs focused on elliptic PDEs and their numerical solutions, a test problem was applied and numerical procedures were used to find their approximate solutions. The results of the different methods were compared and analysed. With important distinctions found among these numerical methods.

References

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