Image Registration

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Introduction

- Visualize objects inside the human body
- Advances in CS methods to diagnosis, treatment planning and medical research
- To diagnosis the integration of data obtained from different sources is desired.

Introduction

- Vital Problem in medical Imaging
- Many potentials Applications in clinical diagnosis:
  - Cardiac
  - Retinal
  - Abdomen
- Process of aligning two images
- Algorithms compute the transformation matrix

Introduction

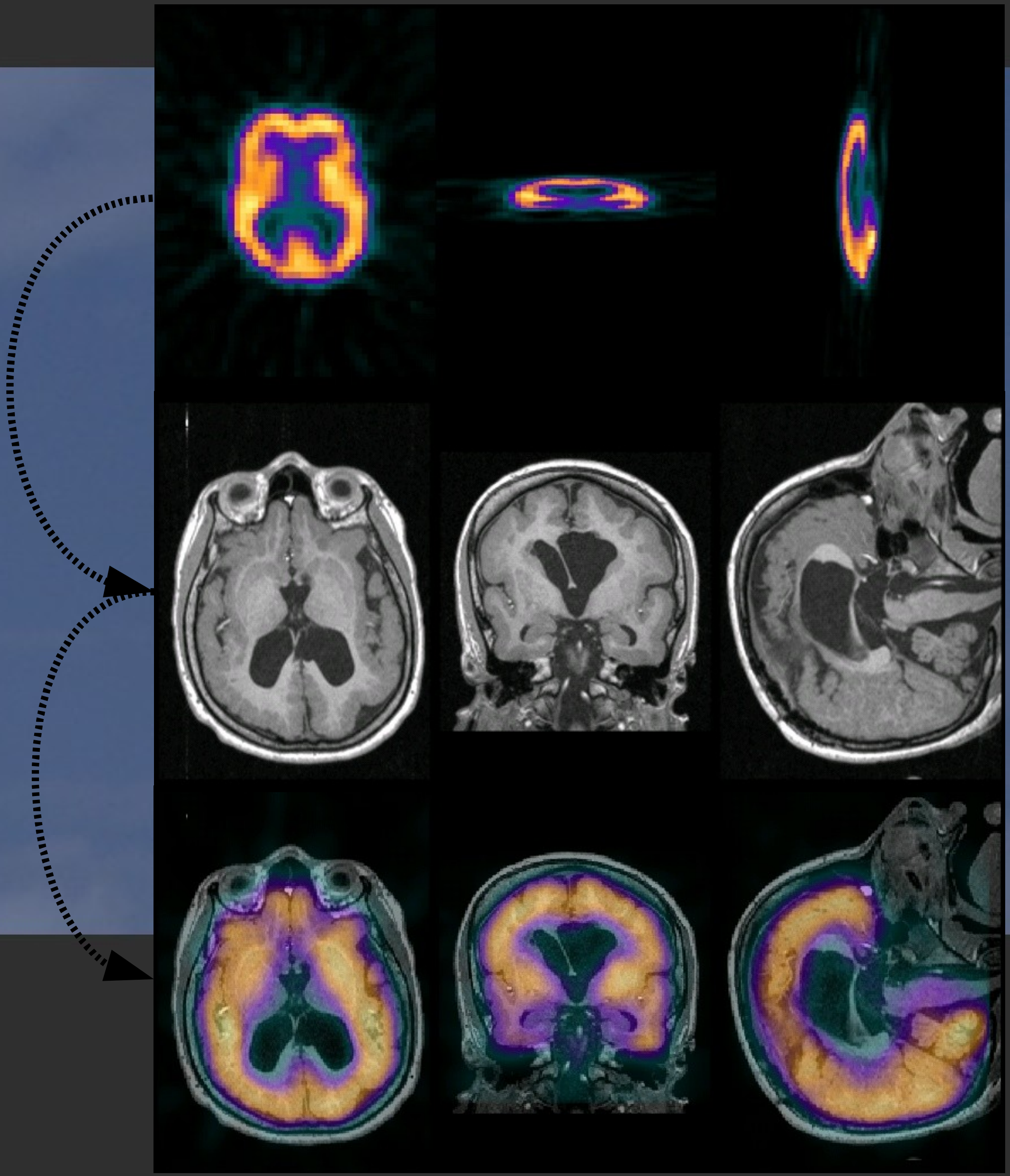
- The target and the source images are different because:
  - Different Times
  - Different source like MRI, CT, PET, SPECT (multi modal)
  - Different angles in order to have a 2D or 3D perspective (multi temporal)

- Classified according the Spatial Transformation:
  - “rigid” (rotation and translation)
  - “non-rigid” (stretching)

Introduction

- Dimensionality Transformations
  - 2D-to-2D
  - 3D-to-3D
  - 2D-to-3D
  - Time (heart cycle and breathing)

Introduction

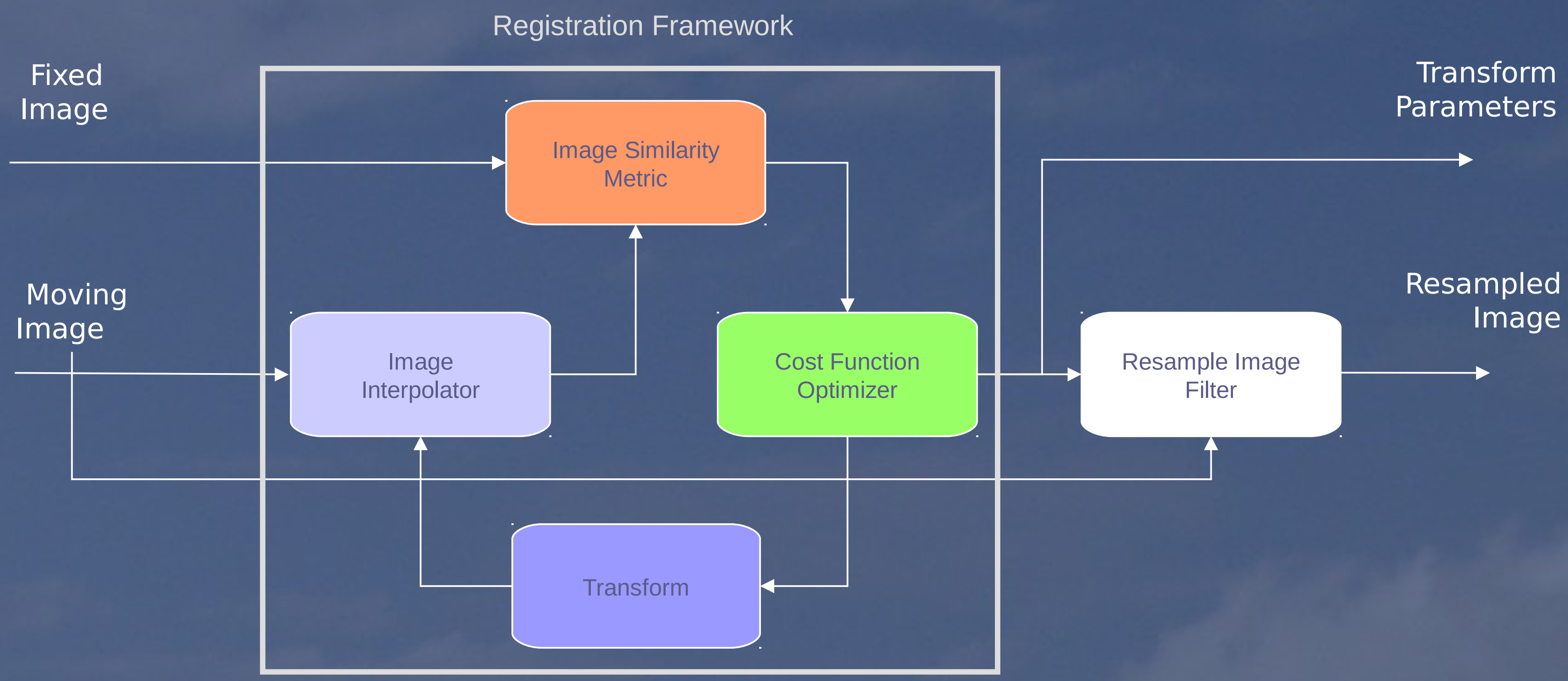


Introduction

- Image Registration Algorithms
  - Landmark-Based Registration
  - Surface-Based Registration
  - Voxel Similarity Measures
  - 2D-3D algorithms
  - Non-rigid algorithms

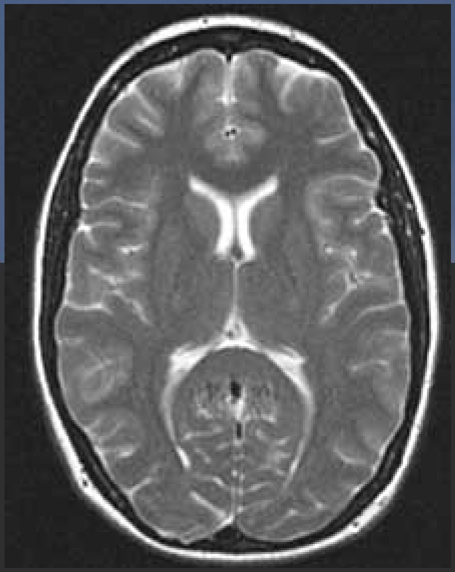
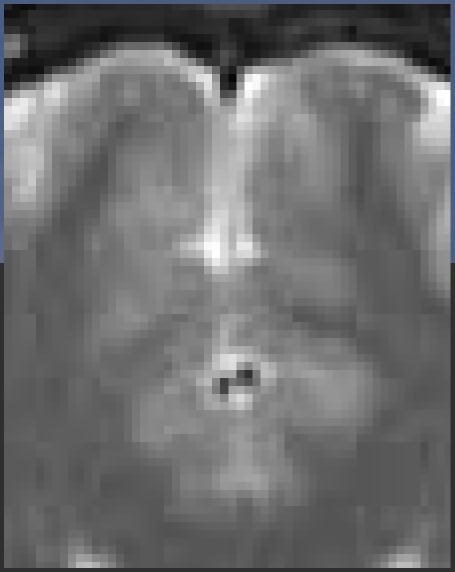
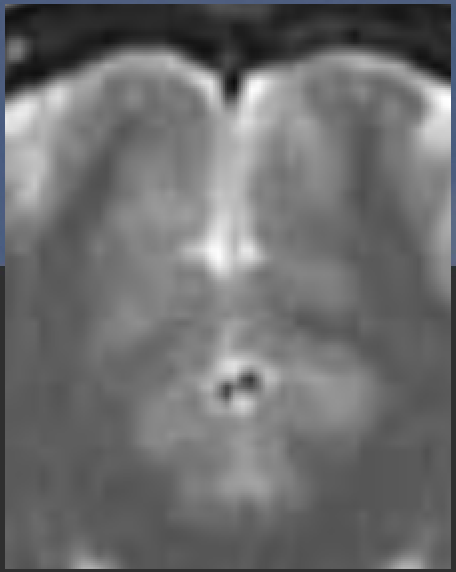
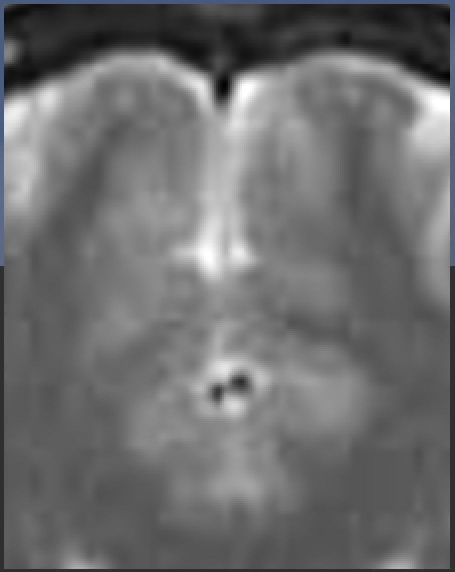
- Optimization

Registration Framework Components (ITK)



Interpolation

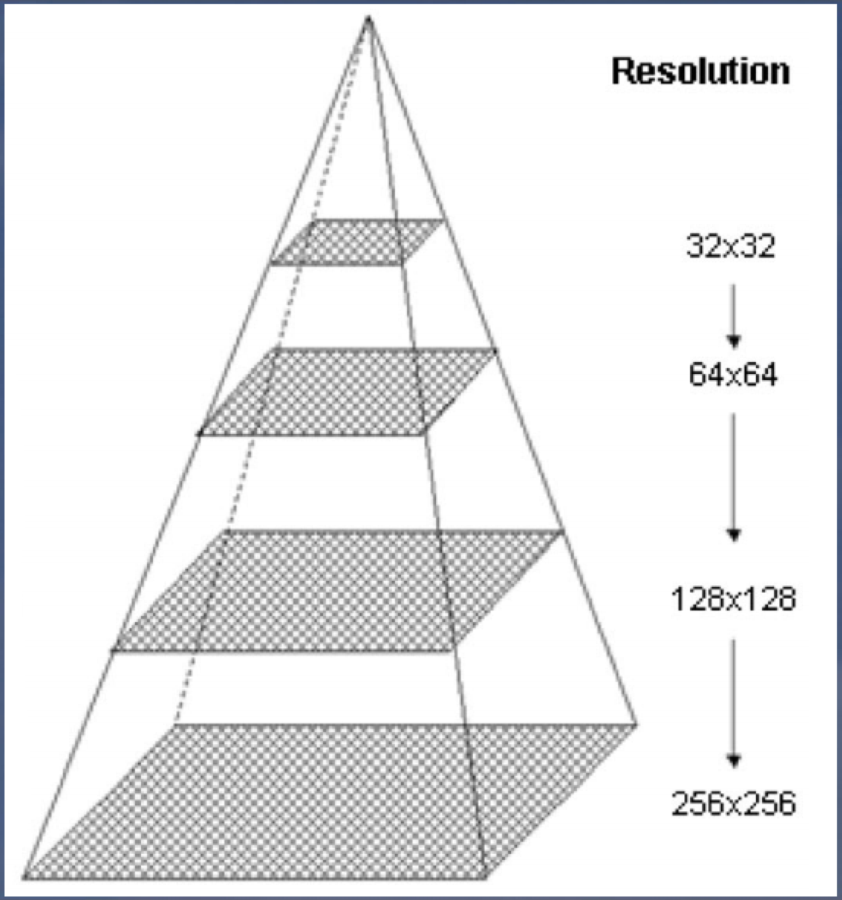
- Lower cost
  - Nearest Neighbor, Linear, Bi-Linear and Cubic
- High cost
  - BSpline, Sync

Concepts and Algorithms

Multi Resolution

- Fast to big changes (small images)
- Accurate in high resolution (complete image)



http://prism.asu.edu/publications/papers/paper04_iruhbs.pdf

Notation

- Refer $A(\mathbf{x}_A)$ as the intensity value in position $\mathbf{x}_A$
- Same to image B
- Images are Mappings of points in the patient within their field of view (or domain Ω) to intensity values

$$A : \mathbf{x}_A \in \Omega_A \mapsto A(\mathbf{x}_A)$$
$$B : \mathbf{x}_B \in \Omega_B \mapsto B(\mathbf{x}_B)$$

Notation

- Images A and B represent one object X
- Imaged with same or different modalities
- There is a relation between spatial locations in A and B
- Registrations involves recovering the spatial transformation T which maps $\mathbf{x}_A$ to $\mathbf{x}_B$

$$\Omega_{A,B}^{\mathbf{T}} = \{\mathbf{x}_A \in \Omega_A \,|\, \mathbf{T}^{-1}(\mathbf{x}_A) \in \Omega_B\}$$

Type of Transformation

- Spatial mapping T describes relationship between images locations
- Same object in a different position (2D-3D)
  - Projection is characterized by system (u_0, v_0, k_u, k_v)
  - u_0, v_0 define the projection of points (u, v) in imaging plane
  - This transformation $T_{projection}$ is 4 x 3 matrix which projects the 3D object along the z axis

$$\mathbf{T}_{projection} = \begin{pmatrix} k_u & 0 & u_0 & 0 \\ 0 & k_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{T}_{2D-3D} = \mathbf{T}_{projection}\mathbf{T}_{rigid}$$

Type of Transformation

- Same object in a different position (3D-3D)
  - Rigid-body transformation (rotation and translation)
  - Six degrees of freedom
    - $t = (t_x, t_y, t_z)$
    - *Rotations about three axes*

$$\mathbf{T}_{rigid}(\mathbf{x}) = \mathbf{R}\mathbf{x} + \mathbf{t}$$

$$\mathbf{T}_{rigid}(\mathbf{x}) = \begin{pmatrix} \cos\beta\cos\gamma & \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma & t_x \\ -\cos\beta\sin\gamma & \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & t_y \\ \sin\beta & -\sin\alpha\cos\beta & \cos\alpha\cos\beta & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Type of Transformation

- Affine or Linear Transformation
  - Rigid-body transformation (rotation and translation)
  - Scaling and Shearing
  - Twelve degrees of freedom

$$\mathbf{T}(x, y, z) = \begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Type of Transformation

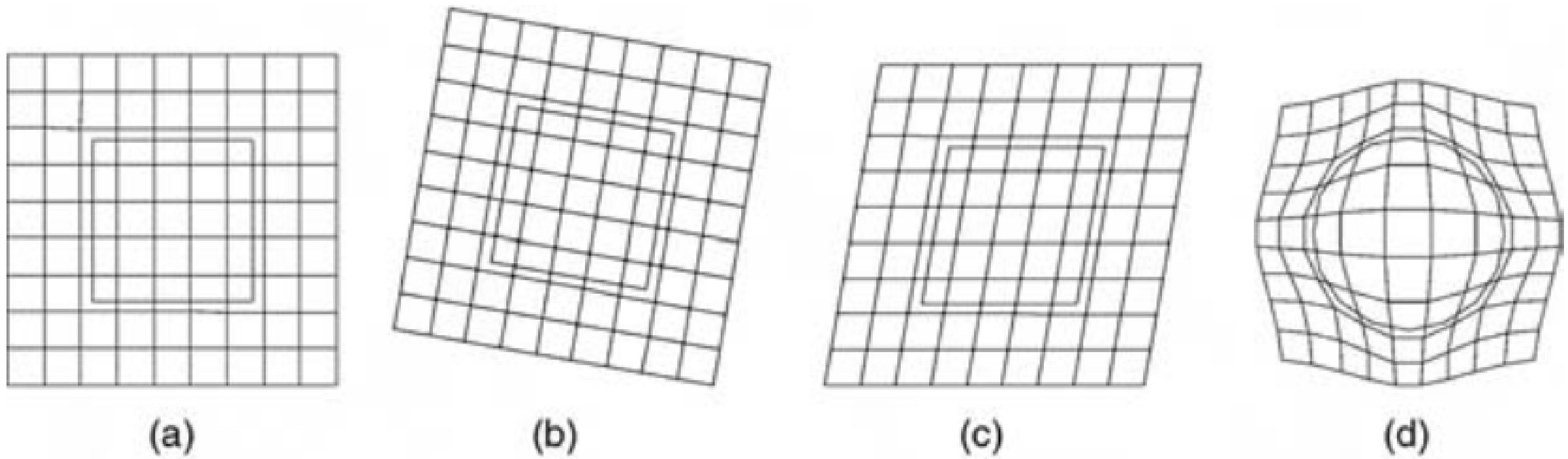


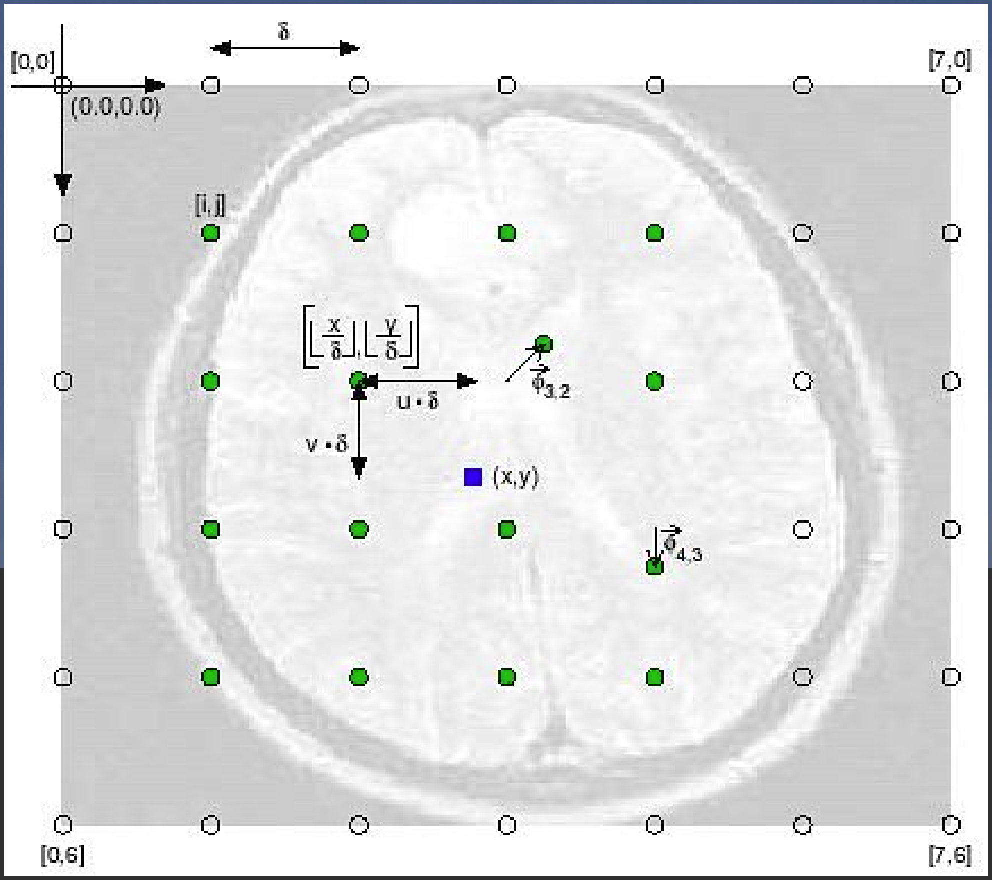
FIGURE 13.1
Example of different types of transformations of a square: (a) identity transformation, (b) rigid transformation, (c) affine transformation, and (d) nonrigid transformation.

Type of Transformation

- Nonrigid Registration
  - Registration Using Basis Functions (Fourier and Wavelets)
  - Thin-Plate Splines
  - B-Splines
  - Elastic Registration
  - Fluid, Optical Flow e etc

Type of Transformation

- **Nonrigid Registration**



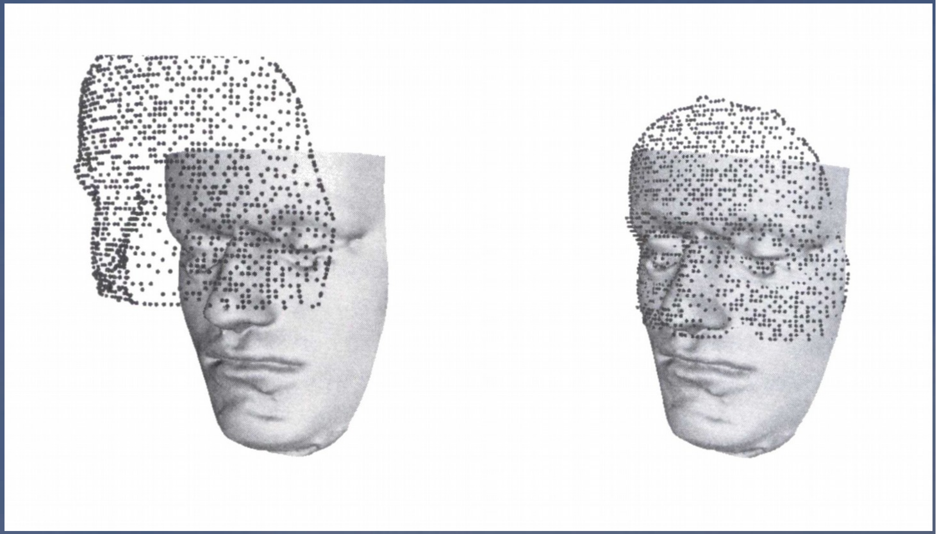
Concepts and Algorithms

Image Spacing and Origin

- Medical image volume are typically anisotropic
  - In-plane pixel size smaller than inter-slice spacing
- A transform is rigid only with respect to physical coordinates and not pixel coordinates
  - PhysCoord = PixelCoord * ImageSpacing + ImageOrigin
- The registration is always with respect to physical coordinates
- Spacing and origin information could be set correctly in the images!

Registration Algorithms

- **Points and the Procrustes Problem**
- **Surface Matching**
  - **The Head and Hat**
  - **Distance Transforms**
  - **Iterative Closest Point (ICP)**
- **Voxel Similarity Measure**



“Handbook of Medical Imaging – Vol. 2. Medical Image Processing and Analysis”, *Editors* Sonka, M.; Fitzpatrick, J.M. - Pag. 479

Voxel Similarity Measure

- Intramodality Registration
  - Minimizing Intensity Difference

$$\mathrm{SSD} = \frac{1}{N} \sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} \left| A(\mathbf{x}_A) - B^{\mathcal{T}}(\mathbf{x}_A) \right|^2$$

$$\mathrm{SAD} = \frac{1}{N} \sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} \left| A(\mathbf{x}_A) - B^{\mathcal{T}}(\mathbf{x}_A) \right|$$

  - Correlation Techniques

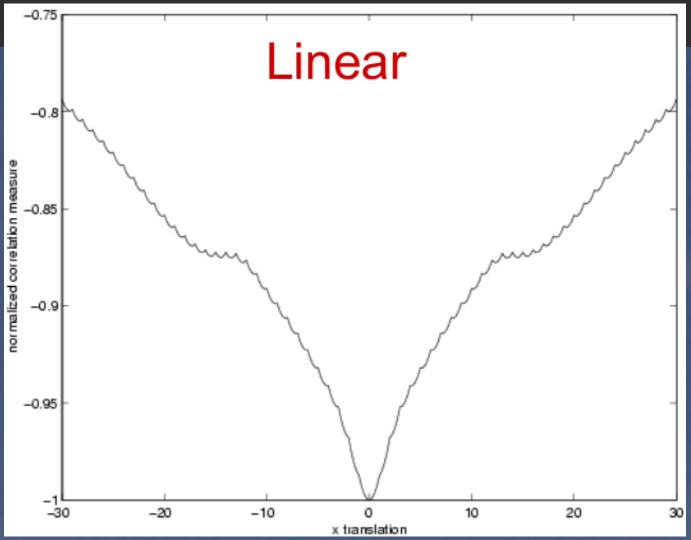
$$\mathrm{CC} = \frac{\sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} (A(\mathbf{x}_A) - \bar{A}) \cdot (B^{\mathcal{T}}(\mathbf{x}_A) - \bar{B})}{\left\{ \sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} (A(\mathbf{x}_A) - \bar{A})^2 \cdot \sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} (B^{\mathcal{T}}(\mathbf{x}_A) - \bar{B})^2 \right\}^{1/2}}$$

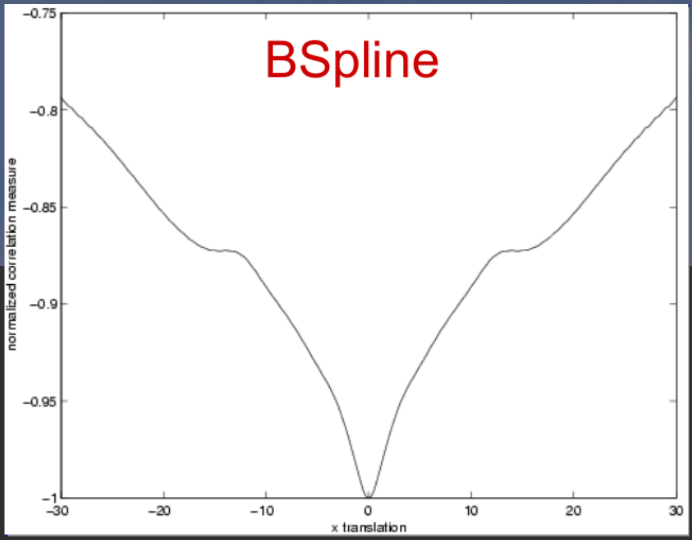
$$\mathrm{C} = \frac{1}{N} \sum_{\mathbf{x}_A \in \Omega^{\mathbf{T}}_{A,B}} A(\mathbf{x}_A) \cdot B^{\mathcal{T}}(\mathbf{x}_A)$$

Voxel Similarity Measure

- Intramodality Registration
  - Normalized Correlation (ITK)
    - Optimal Value -1
    - Metric range: 1 to -1

$$S(\mathbf{p}|F,M,\mathbf{T}) = -1 \times \frac{\sum_i^N F(\mathbf{x}_i) M(\mathbf{x}_i')}{\sqrt{\sum_i^N F^2(\mathbf{x}_i) \sum_i^N M^2(\mathbf{x}_i')}}$$





Voxel Similarity Measure

- Entropy

$$H = -\sum_i p_i \log p_i$$

- Joint Entropy

$$H(A,B) \leq H(A) + H(B)$$

$$H(A,B) = -\sum_a \sum_b p_{AB}^{\mathcal{T}}(a,b) \log p_{AB}^{\mathcal{T}}(a,b)$$

Concepts and Algorithms

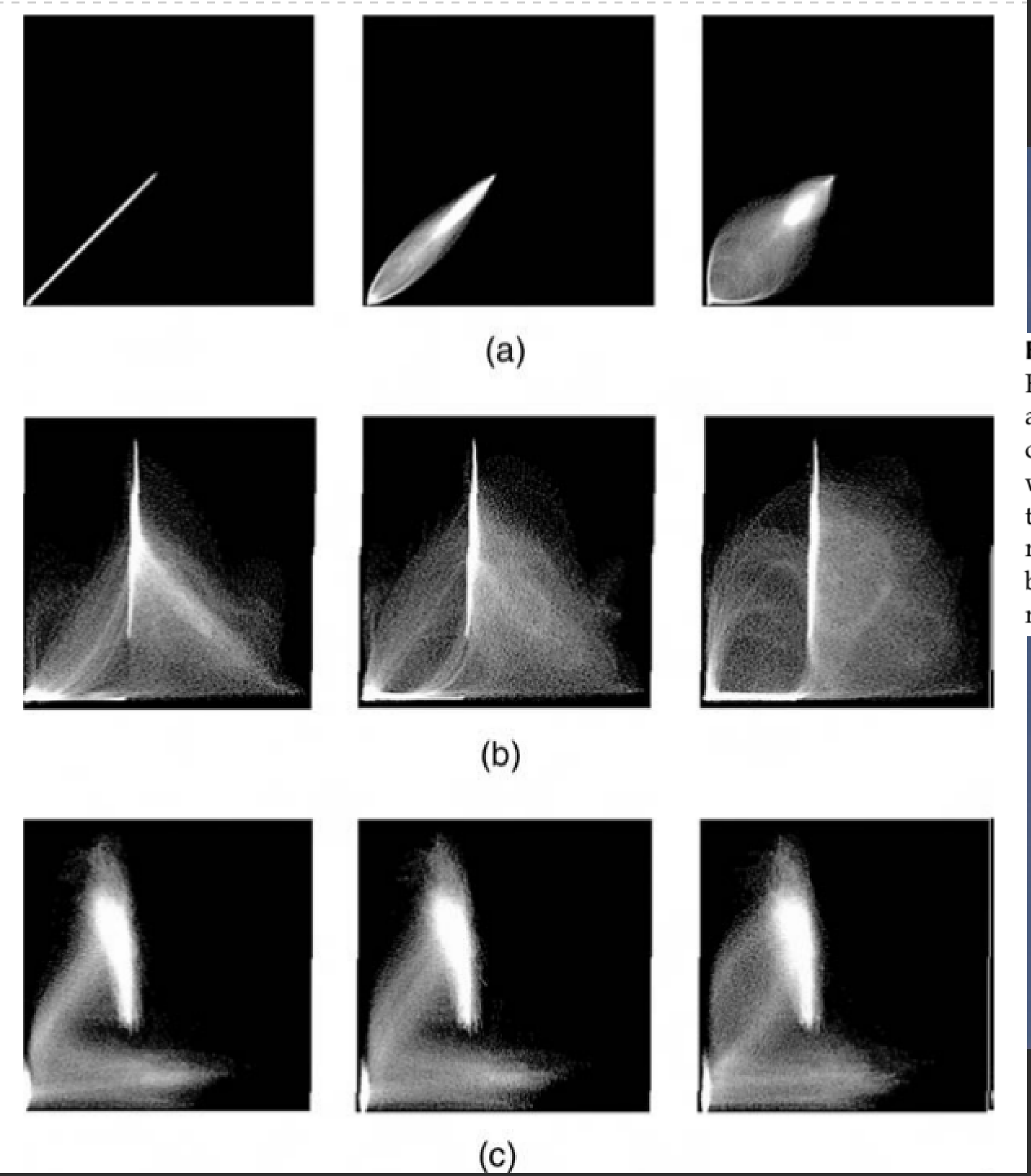


FIGURE 3.1
Example 2D histograms from Hill et al.[43] for (a) identical MR images of the head, (b) MR and CT images of the head, and (c) MR and PET images of the head. For all modality combinations, the left panel is generated from the images when aligned, the middle panel when translated by 2 mm, and the right panel when translated by 5 mm. Note that, while the histograms are quite different for the different modality combinations, misregistration results in a dispersion or blurring of the signal. Although these histograms are generated by lateral translational misregistration, misregistration in other translation or rotation directions has a similar effect.

Voxel Similarity Measure

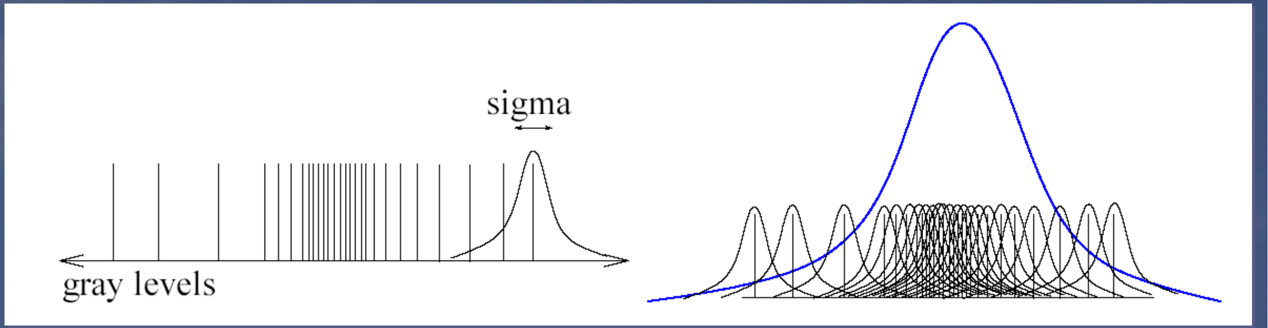
- Intermodality Registration
  - Information theoretic Techniques
  - *Mutual Information Metric:*
    - Qualitatively measure how much information is gained about one image (intensity) by the knowledge of another image (intensity).
    - Two different groups introduced the idea in the context of multi-modality registration. Viola and Well(1997) and Collignon et al. (1995).

Concepts and Algorithms

Voxel Similarity Measure

- Intermodality Registration
  - Information theoretic Techniques
  - *Mutual Information Metric:*
    - If dependency between two images does not have to be specified MI is a good choice.
    - MI is defined by entropy (measure of information)
    - Adjust the bins (32, 64 and etc)
    - Parzen Windowing, Kernel density estimate and Histogram binning.

Parzen Windowing



- Density function is constructed by superimposing kernel functions centered on the intensity samples obtained from the image
- Kernel type
  - Gaussian, boxcar, B-Spline
- Kernel Width (crucial)
  - Depend on dynamic range of data
  - Normalize Data
- Number of Samples

Voxel Similarity Measure

- **Intermodality Registration**
  - **Mutual Information**

$$H(A) = -\sum_{a} p_A^{\mathbf{T}}(a) \log p_A^{\mathbf{T}}(a)$$

$$H(B) = -\sum_{b} p_B^{\mathcal{T}}(b) \log p_B^{\mathcal{T}}(b)$$

$$I(A, B) = H(A) + H(B) - H(A, B) = \sum_{a} \sum_{b} p_{AB}^{\mathcal{T}}(a, b) \log \frac{p_{AB}^{\mathcal{T}}(a, b)}{p_A^{\mathbf{T}}(a) \cdot p_B^{\mathcal{T}}(b)}$$

Voxel Similarity Measure

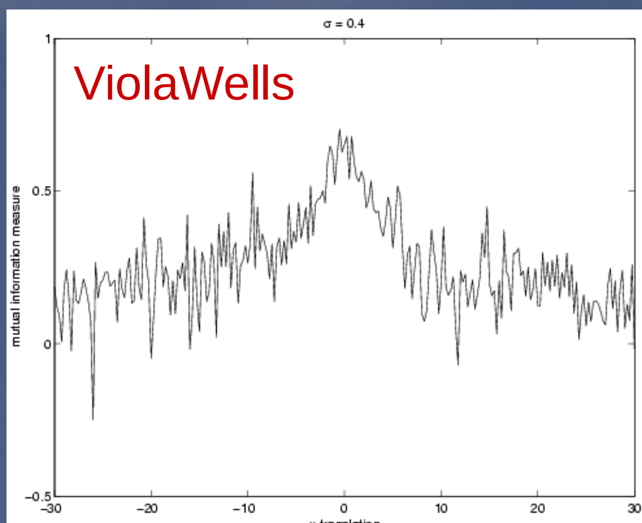
- Intermodality Registration
  - Normalized Mutual Information

$$\tilde{I}_1(A,B) = \frac{2I(A,B)}{H(A)+H(B)}$$

$$\tilde{I}_2(A,B) = H(A,B) - I(A,B)$$

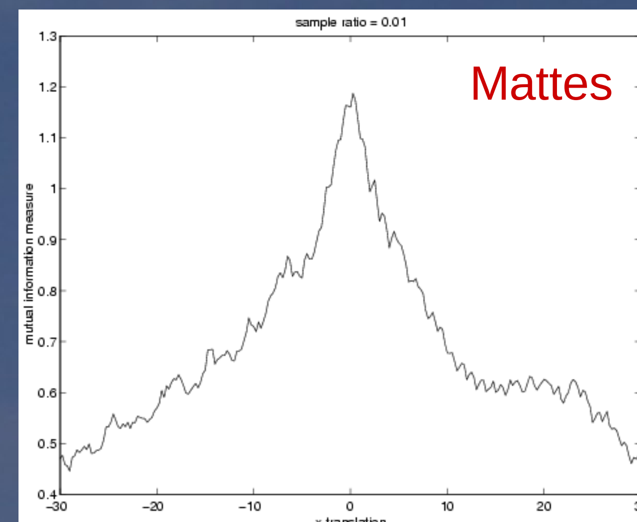
$$\tilde{I}_3(A,B) = \frac{H(A)+H(B)}{H(A,B)}$$

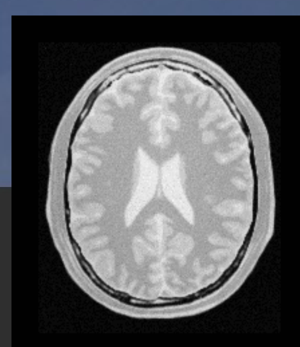
Mutual Information Metric

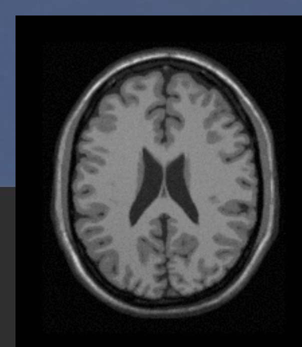


Optimal Value at maximum

Metric range: Image dependent



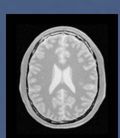


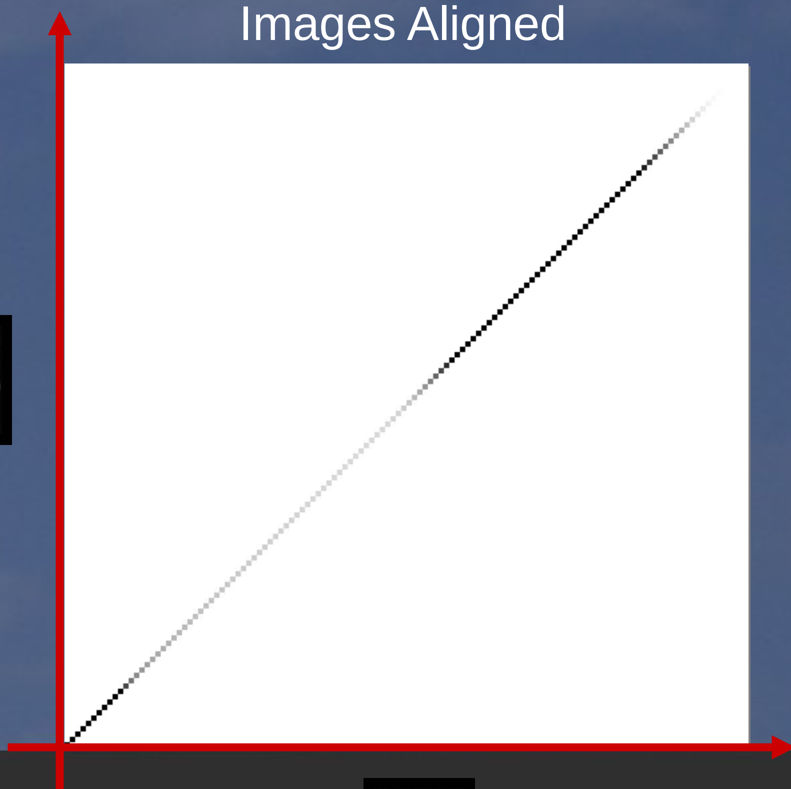


Translations

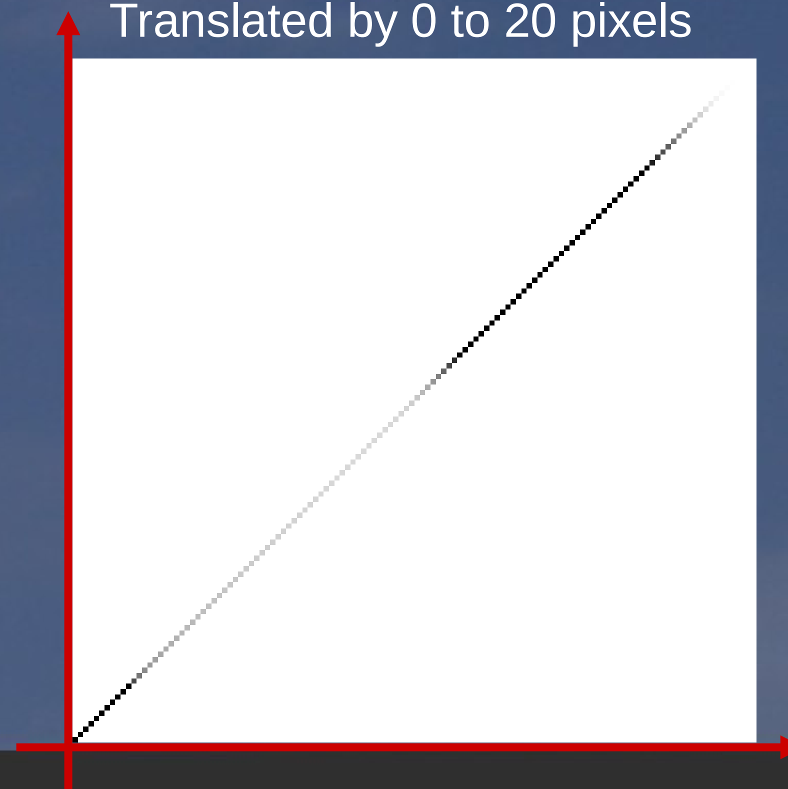
Joint Histograms: Mono-Modality

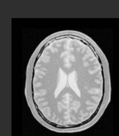
Images Aligned





Translated by 0 to 20 pixels



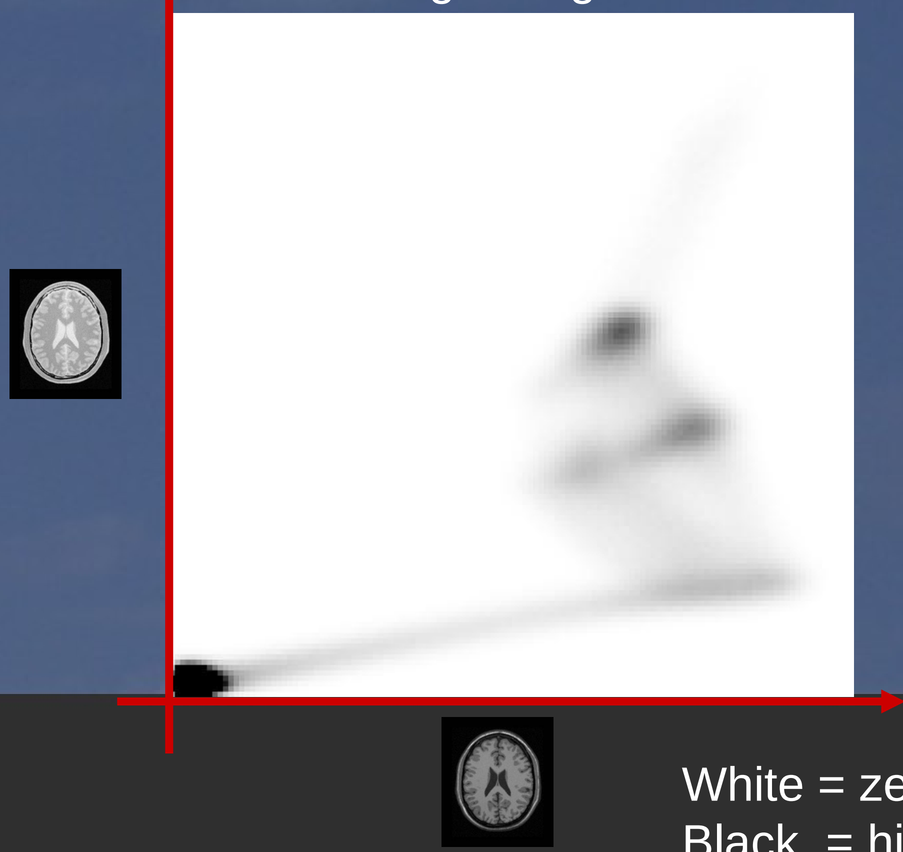


White = zero value
Black = highest value

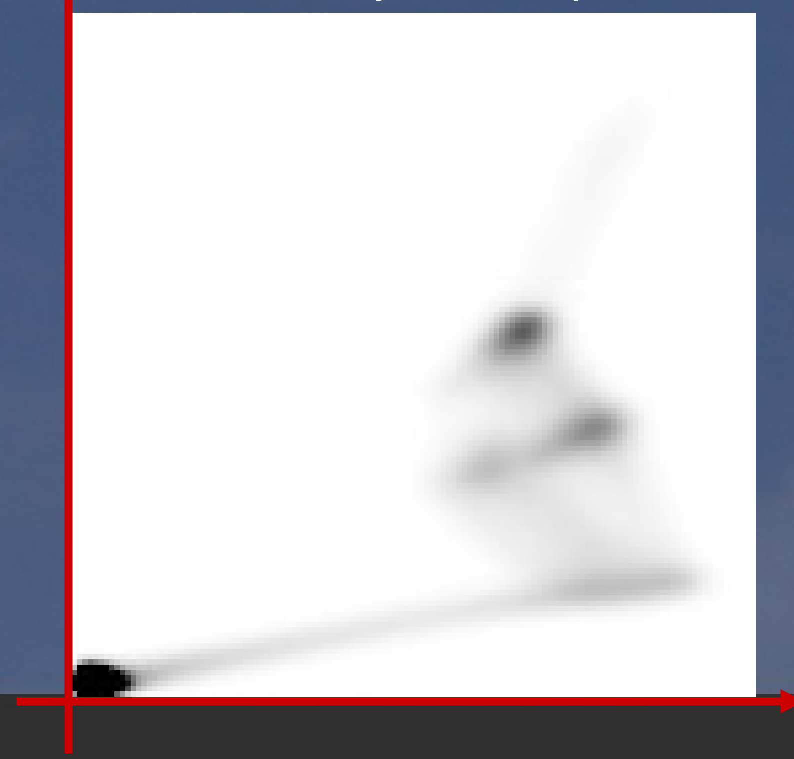
Misalignment causes dispersion

Joint Histograms: Multi-Modality

Images Aligned



Translated by 0 to 20 pixels



White = zero value
Black = highest value

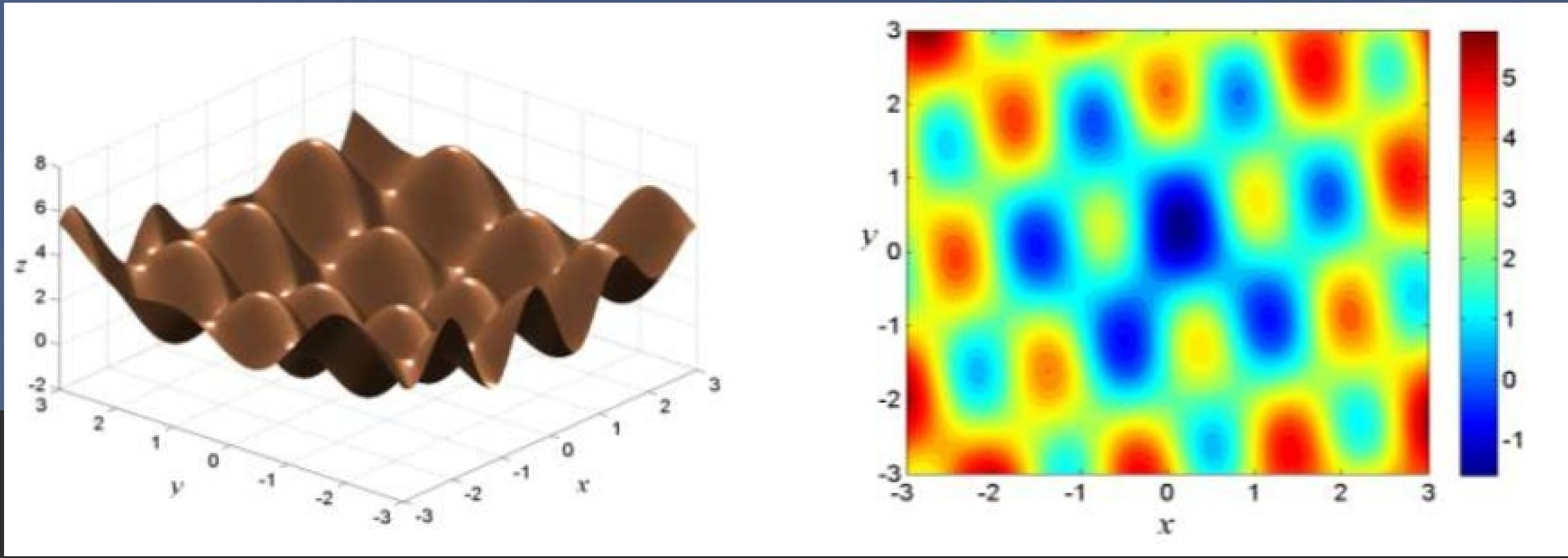
Misalignment causes dispersion

Optimization

- Gradient Descent
- Powell’s Direction Set Method
- Downhill Simplex method

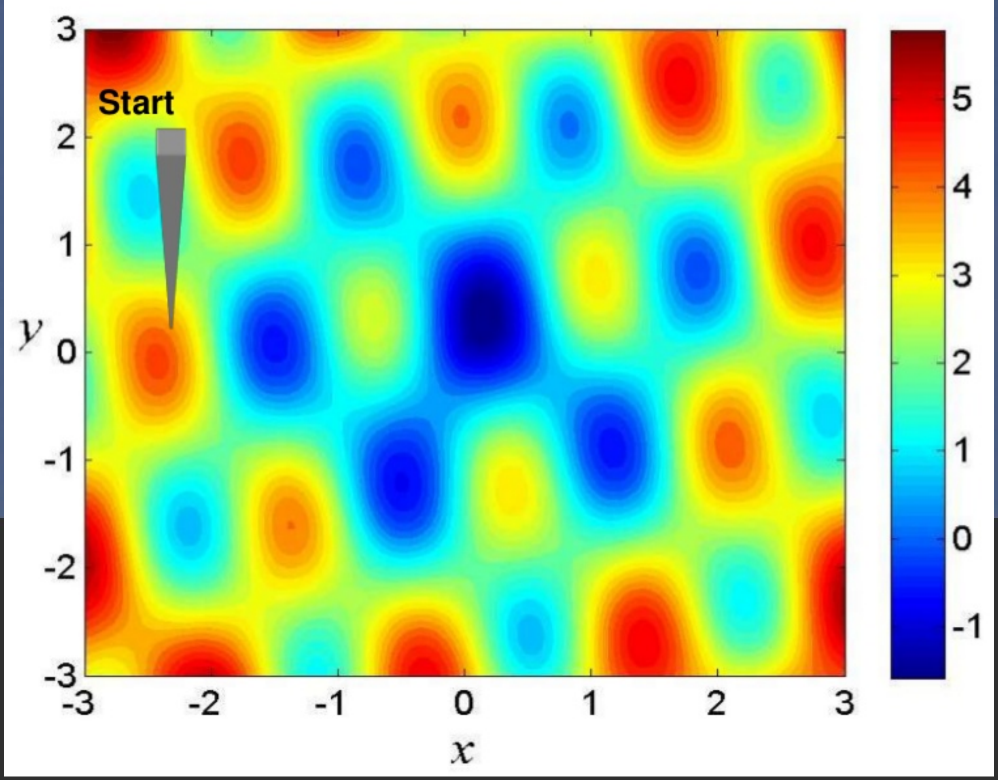
Optimization

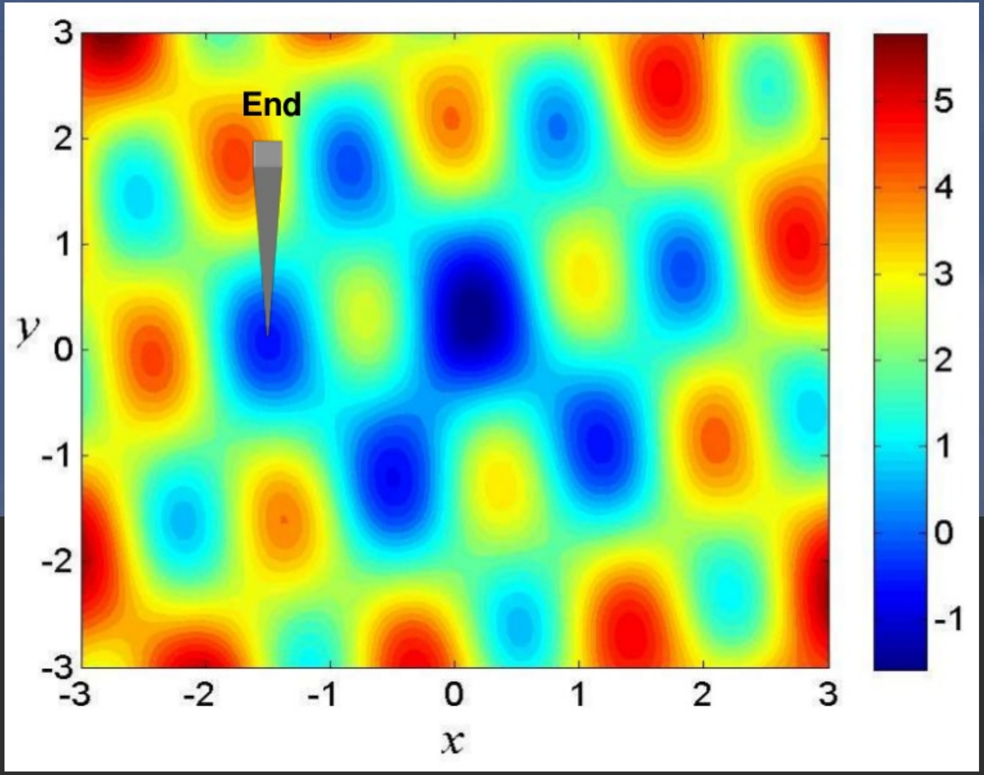
- **Gradient Descent**



Optimization

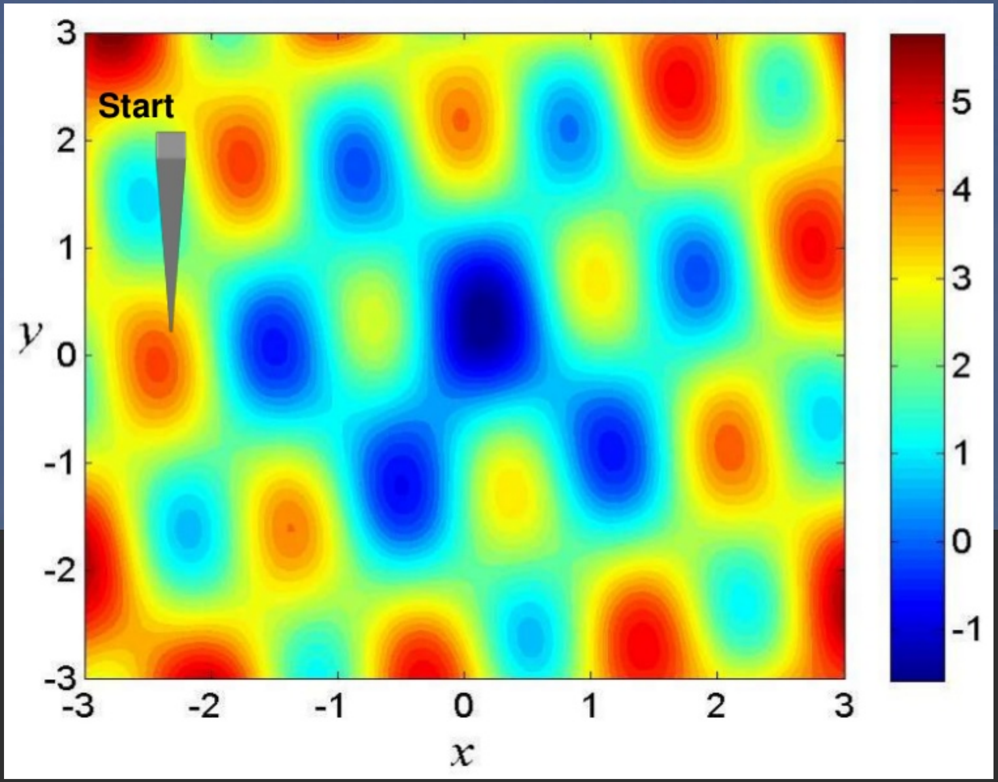
- **Gradient Descent (Local Optimization)**

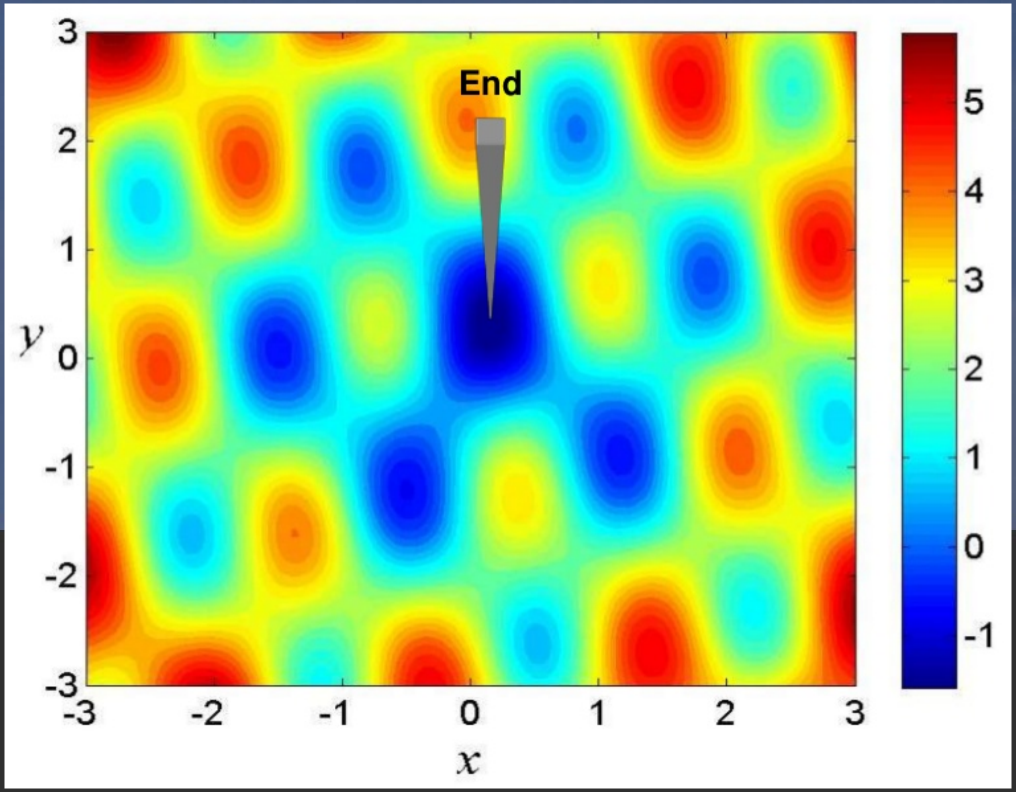




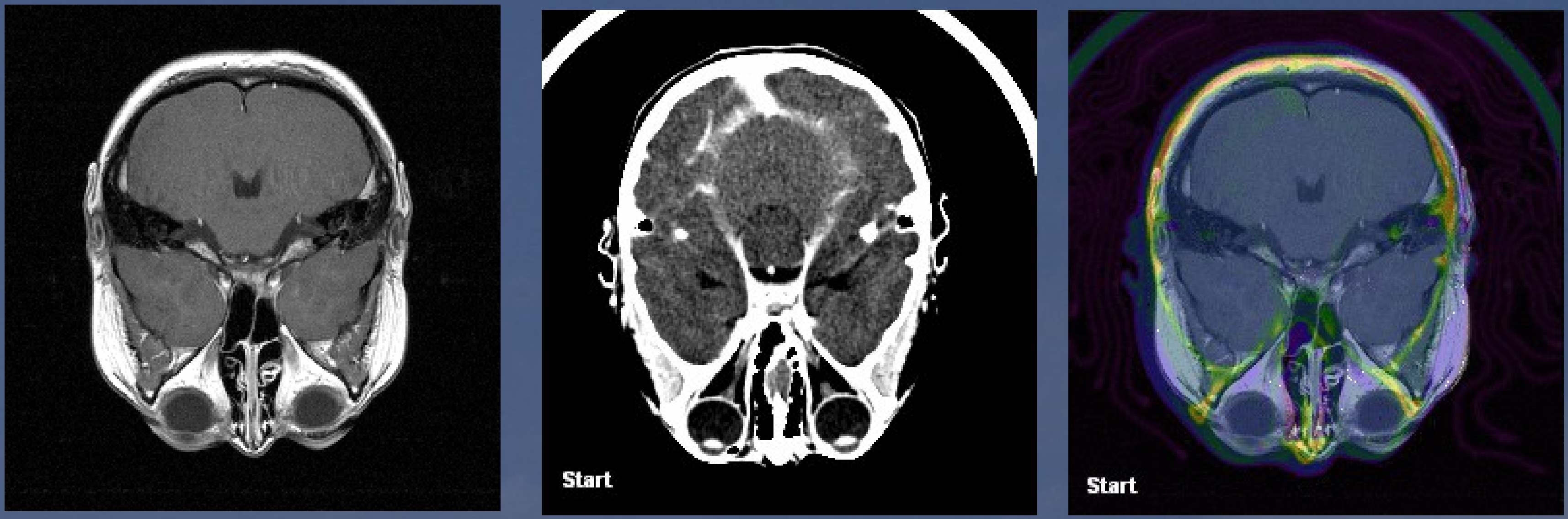
Optimization

- **Gradient Descent (Global Optimization)**





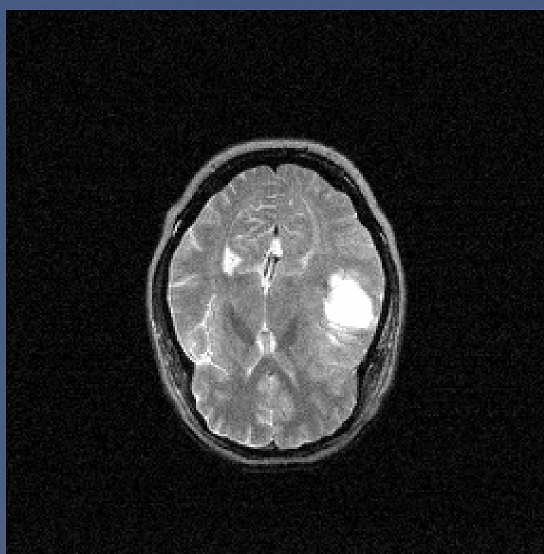
3D CT to MR-T1 Rigid Registration

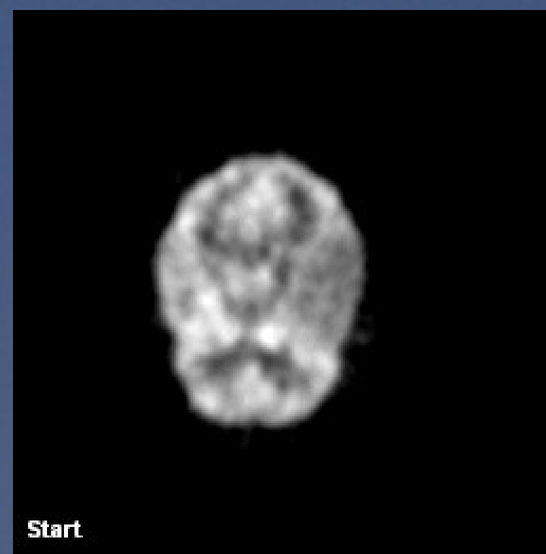


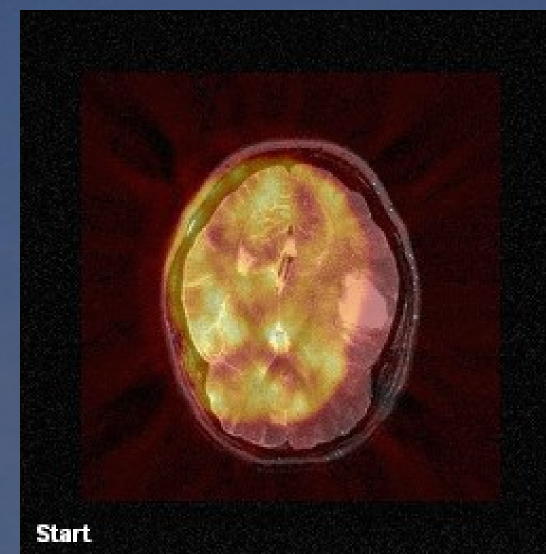
Fixed Image: MR-T1, 256 x 256 x 52 pixels, 0.78 x 0.78 x 3.00 mm
Moving Image: CT, 512 x 512 x 44, 0.41 x 0.41 x 3.00 mm
Registration: 4 levels, MI, gradient descent, quaternion rigid

Images provided as part of the project: “Retrospective Image Registration Evaluation”,
NIH, Project No. 8R01EB002124-03, Principal Investigator, J. Michael Fitzpatrick, Vanderbilt University, Nashville, TN.

3D PET to MR-T2 Rigid Registration







Fixed Image: MR-T2, 256 x 256 x 26 pixels, 1.25 x 1.25 x 4.00 mm
Moving Image: PET, 128 x 128 x 15, 1.94 x 1.94 x 8.00 mm
Registration: 3 levels, MI, gradient descent, quaternion rigid

Images provided as part of the project: “Retrospective Image Registration Evaluation”,
NIH, Project No. 8R01EB002124-03, Principal Investigator, J. Michael Fitzpatrick, Vanderbilt University, Nashville, TN.

References:

Hajnal J.; Hawkes, D. and Hill, D.L.G. (editors) “Medical Image Registration”, CRC Press, The BIOMEDICAL ENGINEERING Series (editor Michael Neuman), 2001, ISBN 0-8493-0064-9

Studholme, C.; Hill, D.L.G.; Hawkes, D.J. “Automated three-dimensional registration of magnetic resonance and positron emission tomography brain images by multiresolution optimization of voxel similarity measures”, Med. Phys. 24(1), pp. 25-35, Jan. 1997

Thévenaz, P.; Unser, M. “Optimization of Mutual Information for Multiresolution Image Registration”, IEEE Transactions on Image Processing, Vol. 9, No. 12, pp. 2083-2099 Dec. 2000

http://pt.slideshare.net/kitware/itk-tutorial-presentation-slides947

http://slideplayer.com/slide/5182766/

http://pt.slideshare.net/balanou/thesis-image-registration-methods

https://www.coursera.org/learn/machine-learning/lecture/kCvQc/gradient-descent-for-linear-regression

Image Registration

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